Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2010

MARKING SCHEME

APPLIED MATHEMATICS

ORDINARY LEVEL

General Guidelines

1. Penalties of three types are applied to candidates' work as follows:

   Slips - numerical slips S(-1)

   Blunders - mathematical errors B(-3)

   Misreading - if not serious M(-1)

   Serious blunder or omission or misreading which oversimplifies:
   - award the attempt mark only.

   Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

2. The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. A car travels along a straight level road.
It passes a point P at a speed of $12\ \text{m s}^{-1}$ and accelerates uniformly for 6 seconds to a speed of $30\ \text{m s}^{-1}$.
It then travels at a constant speed of $30\ \text{m s}^{-1}$ for 15 seconds.
Finally the car decelerates uniformly from $30\ \text{m s}^{-1}$ to rest at a point Q.
The car travels 45 metres while decelerating.

Find **(i)** the acceleration
(ii) the deceleration
(iii) $|PQ|$, the distance from P to Q
(iv) the average speed of the car as it travels from P to Q.

(i)
$$v = u + ft$$
$$30 = 12 + f(6)$$
$$f = 3\ \text{m s}^{-2}$$
10

(ii)
$$v^2 = u^2 + 2fs$$
$$0 = (30)^2 + 2f(45)$$
$$f = -10\ \text{m s}^{-2}$$
10

(iii)
$$s = ut + \tfrac{1}{2}at^2$$
$$s_1 = 12(6) + \tfrac{1}{2}(3)(36)$$
$$s_1 = 126\ \text{m.}$$
10

$$s = ut + \tfrac{1}{2}at^2$$
$$s_2 = 30(15) + 0$$
$$s_2 = 450\ \text{m}$$
5

$$|PQ| = 126 + 450 + 45$$
$$= 621\ \text{m}$$
5

(iv)
$$t_3 = \frac{v-u}{f} = \frac{0-30}{-10} = 3$$
5

$$\text{average speed} = \frac{\text{distance}}{\text{time}}$$
$$= \frac{621}{6+15+3}$$
$$= 25.875\ \text{m s}^{-1}$$
5

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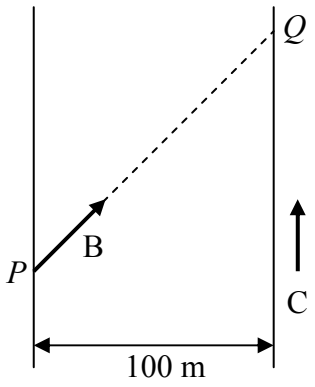
2. A river is 100 metres wide and has parallel banks.



Boat B departs from point P on its western bank and lands at point Q on its eastern bank.

The actual velocity of the boat is $5\,\vec{i} + 12\,\vec{j}$ m s^{-1}.

Cyclist C travels due north at a constant speed of 3 m s^{-1} along the eastern bank of the river.

Find **(i)** the velocity of C in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of B relative to C in terms of $\vec{i}$ and $\vec{j}$

(iii) the magnitude and direction of the velocity of B relative to C

(iv) the time it takes B to cross the river

(v) $|PQ|$, the distance from P to Q.

(i) $\vec{V}_C = 0\,\vec{i} + 3\,\vec{j}$ — 10

(ii) $\vec{V}_{BC} = \vec{V}_B - \vec{V}_C$ — 5

$= (5\,\vec{i} + 12\,\vec{j}) - (0\,\vec{i} + 3\,\vec{j})$

$= 5\,\vec{i} + 9\,\vec{j}$ — 5

(iii) $|\vec{V}_{BC}| = \sqrt{5^2 + 9^2}$

$= \sqrt{106}$ *or* 10.3 — 5

dirn = E 60.9° N. — 5

(iv) $\text{time} = \frac{100}{5} = 20\text{ s}$ — 10

(v) speed along $PQ = \sqrt{5^2 + 12^2}$

$= 13$ m s^{-1} — 5

$|PQ| = 20(13)$

$= 260$ m — 5

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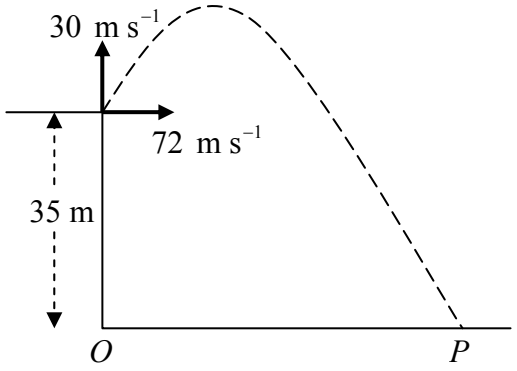
3. A particle is projected with initial velocity $72\ \vec{i} + 30\ \vec{j}$ m s^{-1} from the top of a straight vertical cliff of height 35 m. It strikes the horizontal ground at P.



Find

(i) the time taken to reach the maximum height

(ii) the maximum height of the particle above ground level

(iii) the time of flight

(iv) $|OP|$, the distance from O to P

(v) the speed of the particle as it strikes the ground.

(i)
$$v = u + ft$$
$$0 = 30 - 10t$$
$$t = 3 \text{ s} \qquad 10$$

(ii)
$$s = ut + \tfrac{1}{2} f t^2$$
$$= 30(3) - 5(9)$$
$$= 45 \text{ m}$$
$$\text{distance} = 45 + 35 = 80 \text{ m} \qquad 10$$

(iii)
$$s_y = ut + \tfrac{1}{2} a t^2$$
$$-35 = 30(t) - 5t^2$$
$$t^2 - 6t - 7 = 0$$
$$t = 7 \text{ s} \qquad 10$$

(iv)
$$|OP| = ut + \tfrac{1}{2} a t^2$$
$$= 72(7) - 0$$
$$= 504 \text{ m} \qquad 10$$

(v)
$$v_y = u + at$$
$$= 30 - 10(7)$$
$$= -40 \qquad 5$$

$$v = \sqrt{72^2 + (-40)^2}$$
$$= 82.4 \text{ m s}^{-1} \qquad 5$$

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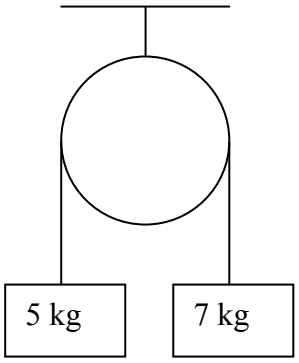
4. **(a)** Two particles of masses 5 kg and 7 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley.



The system is released from rest.

Find **(i)** the common acceleration of the particles

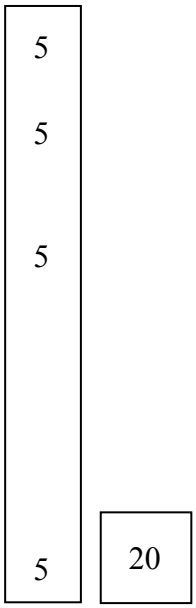
(ii) the tension in the string.

(i)

$$T - 5g = 5a$$

$$7g - T = 7a$$

$$a = \frac{20}{12} \text{ or } \frac{5}{3} \text{ m s}^{-2}$$



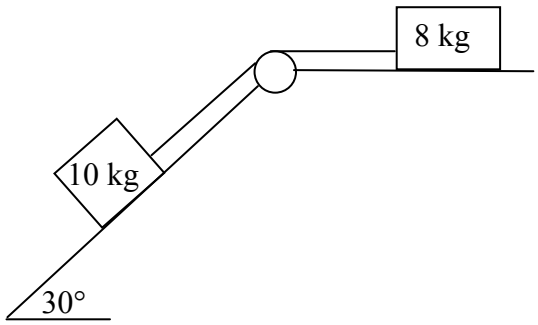
(ii)

$$T = 5a + 5g$$

$$T = \frac{25}{3} + 50$$

$$= 58.3 \text{ N}$$

4 **(b)** Masses of 8 kg and 10 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley as shown in the diagram.



The 8 kg mass lies on a rough horizontal plane and the coefficient of friction between the 8 kg mass and the plane is $\frac{1}{2}$.

The 10 kg mass lies on a smooth plane which is inclined at 30° to the horizontal.

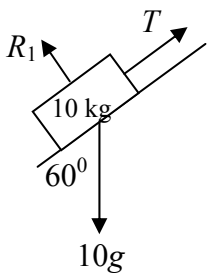
The system is released from rest.

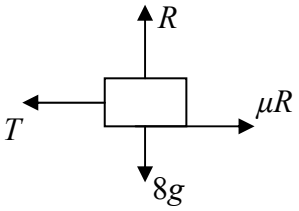
(i) Show on separate diagrams the forces acting on each particle.

(ii) Find the common acceleration of the masses.

(iii) Find the tension in the string.

(i)





10

(ii)

$$T - \mu R = 8a$$
$$T - \tfrac{1}{2}(8g) = 8a$$

5

$$10g\cos 60 - T = 10a$$
$$50 - T = 10a$$

5

$$50 - 40 = 18a$$
$$a = \frac{10}{18} \text{ or } \frac{5}{9} \text{ m s}^{-2}$$

5

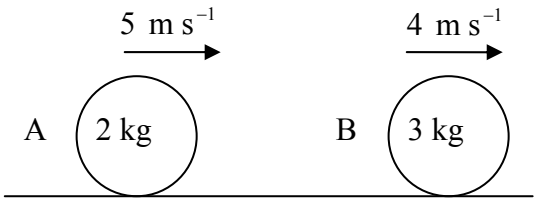
(iii)

$$T = 50 - 10a$$
$$= 44.4 \text{ N}$$

5

30

5. A smooth sphere A, of mass 2 kg, collides directly with another smooth sphere B, of mass 3 kg, on a smooth horizontal table.



A and B are moving in the same direction with speeds of $5\ \text{m s}^{-1}$ and $4\ \text{m s}^{-1}$ respectively.

The coefficient of restitution for the collision is $\frac{2}{3}$.

Find **(i)** the speed of A and the speed of B after the collision

(ii) the change in the kinetic energy of A due to the collision

(iii) the magnitude of the impulse imparted to A due to the collision.

(i) PCM $\quad 2(5) + 3(4) = 2v_1 + 3(v_2)$ — 10

$22 = 2v_1 + 3v_2$

NEL $\quad v_1 - v_2 = -e(u_1 - u_2)$

$= -\frac{2}{3}(5-4)$ — 10

$= -\frac{2}{3}$

$v_1 = 4\ \text{m s}^{-1}$ and $v_2 = \frac{14}{3}\ \text{m s}^{-1}$ — 10

(ii) KE before collision $= \frac{1}{2}(2)(5)^2$

$= 25$ — 5

KE after collision $= \frac{1}{2}(2)(4)^2$

$= 16$ — 5

Change in KE of A $= 25 - 16$

$= 9\ \text{J}$ — 5

(iii) Impulse $= |(2)(4) - (2)(5)|$

$= 2\ \text{N s}$ — 5

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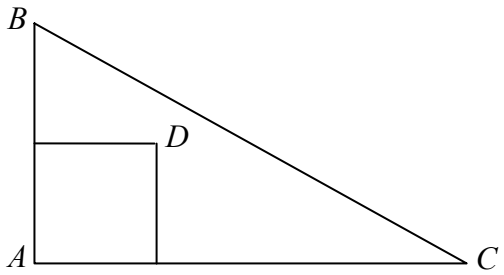
6. **(a)** Particles of weight 3 N, 7 N, 1 N and 5 N are placed at the points $(1, p)$, $(p, 2)$, $(-2, q)$ and $(4, 8)$, respectively.
The co-ordinates of the centre of gravity of the system are $(3\cdot 5, 4\cdot 5)$.

Find **(i)** the value of p

(ii) the value of q.

(b) A triangular lamina with vertices A, B and C has the square portion with diagonal $[AD]$ removed.

The co-ordinates of the points are $A(0,0)$, $B(0,9)$, $C(12,0)$ and $D(4,4)$.



Find the co-ordinates of the centre of gravity of the remaining lamina.

(a)
$$3.5 = \frac{3(1)+7(p)+1(-2)+5(4)}{16} \qquad 10$$
$$p = 5 \qquad 5$$
$$4.5 = \frac{3(p)+7(2)+1(q)+5(8)}{16} \qquad 10$$
$$q = 3 \qquad 5$$

(b)

	*a*rea :	c.g.	
square	$(4)(4)=16$	$(2,2)$	5
ABC	$\frac{1}{2}(12)(9)=54$	$(4,3)$	5
lamina	$=38$	(x, y)	

$$(38)(x) = 54(4) - 16(2) \qquad 5$$
$$x = 4.8$$
$$(38)(y) = 54(3) - 16(2) \qquad 5$$
$$y = 3.4$$

co - ords of c.g. $(4.8, 3.4)$

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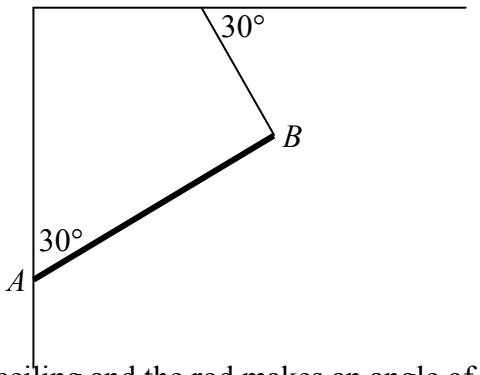
7. A uniform rod, $[AB]$, of length 2 m and weight 40 N is smoothly hinged at end A to a vertical wall.

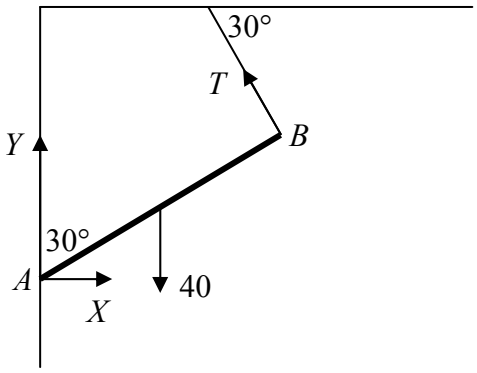
One end of a light inelastic string is attached to B and the other end of the string is attached to a horizontal ceiling.



The string makes an angle of $30°$ with the ceiling and the rod makes an angle of $30°$ with the wall, as shown in the diagram.

The rod is in equilibrium.

(i) Show on a diagram all the forces acting on the rod $[AB]$.
(ii) Write down the two equations that arise from resolving the forces horizontally and vertically.
(iii) Write down the equation that arises from taking moments about point A.
(iv) Find the tension in the string.
(v) Find the magnitude of the reaction at the hinge, A.



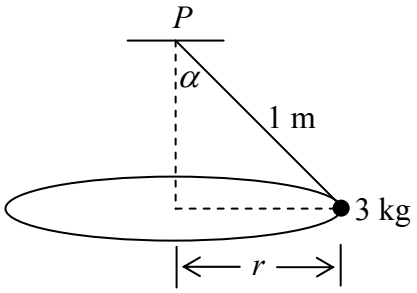
		10
horiz	$X = T\cos 30$	5
vert	$Y + T\sin 30 = 40$	5
Take moments about A :		
	$T(2) = 40(1\sin 30)$	10
	$T = 10$ N	5
	$X = T\cos 30 = 5\sqrt{3}$	5
	$Y + T\sin 30 = 40 \Rightarrow y = 35$	5
	$R = \sqrt{(5\sqrt{3})^2 + 35^2}$	
	$= 36.1$ N	5
		50

8. **(a)** A particle describes a horizontal circle of radius r metres with uniform angular velocity ω radians per second.
Its speed and acceleration are $6\ \text{m s}^{-1}$ and $12\ \text{m s}^{-2}$ respectively.

Find **(i)** the value of r
(ii) the value of ω .

(b) A conical pendulum consists of a particle of mass 3 kg attached by a light inelastic string of length 1 metre to a fixed point P.

The particle describes a horizontal circle of radius r.
The centre of the circle is vertically below P.
The string makes an angle of α with the vertical where $\tan\alpha = \frac{4}{3}$.



Find **(i)** the value of r
(ii) the tension in the string
(iii) the angular velocity of the particle.

(a)

$$\text{speed} = r\omega$$
$$6 = r\omega \qquad 5$$
$$\text{acceleration} = r\omega^2$$
$$12 = r\omega^2 = \omega(r\omega) \qquad 5$$
$$\Rightarrow \quad \omega = 2\ \text{rad s}^{-1} \qquad 5$$

$$6 = r\omega$$
$$\Rightarrow r = 3\ \text{m} \qquad 5$$

(b)

(*i*) $\tan\alpha = \frac{4}{3} \Rightarrow \sin\alpha = \frac{4}{5}$ 5

$\frac{r}{1} = \frac{4}{5} \quad \Rightarrow r = 0.8\ \text{m}$ 5

(*ii*) $T\cos\alpha = 3g$ 5

$T(0.6) = 30 \ \Rightarrow T = 50\ \text{N}$ 5

(*iii*) $T\sin\alpha = mr\omega^2$ 5

$50(0.8) = 3(0.8)\omega^2$

$\Rightarrow \ \omega = 4.08\ \text{rad s}^{-1}$ 5

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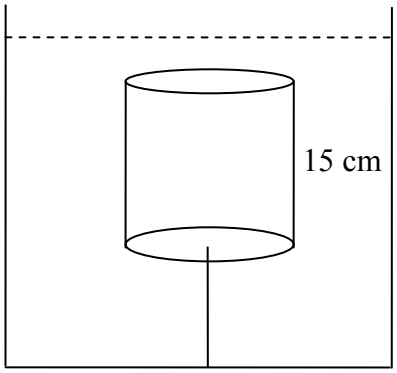
9. **(a)** State the Principle of Archimedes.

A solid piece of metal has a weight of 14 N.
When it is completely immersed in water the metal weighs 9 N.

Find **(i)** the volume of the metal
(ii) the relative density of the metal.

(b) A right circular solid cylinder has a base of radius 6 cm and a height of 15 cm.



The relative density of the cylinder is 0·7 and it is completely immersed in a tank of liquid of relative density 0·9.

The cylinder is held at rest by a light inextensible vertical string which is attached to the base of the tank. The upper surface of the cylinder is horizontal.

Find the tension in the string.

[Density of water = 1000 kg m^{-3}]

(a)

Principle of Archimedes: 10

(i) B = weight of water displaced
$5 = 1000V(10)$
$\Rightarrow V = 5\times10^{-4}$ m^3 10

(ii) weight of metal $= \rho Vg$
$14 = \rho\left(5\times10^{-4}\right)(10)$
$\rho = 2800 \quad \Rightarrow s = 2.8$ 10

(b)

$B = 900\left\{\pi\times(0.06)^2\times(0.15)\right\}(10)$
$= 15.27$ 5

$W = 700\left\{\pi\times(0.06)^2\times(0.15)\right\}(10)$
$= 11.88$ 5

$T + W = B$ 5

$T = 15.27 - 11.88$
$= 3.39$ N 5

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Marcanna Breise as ucht freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc × 5% = 9·9 $\Rightarrow$ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] × 15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0